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Can the Solar Cycle Amplitude Be Predicted Using the Preceding Solar Cycle Length?

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Abstract In this work, the evolution of the relationship between Solar Cycle Length of solar cycle n (SCL_n) and Solar Cycle Amplitude of the solar cycle $n+1$ (SCA_{n+1}) is studied by using the R_{Z} and R_{G} sunspot numbers. We conclude that this relationship is only strongly significant in a statistical sense during the first half of the historical record of R_{Z} sunspot number whereas it is considerably less significant for the R_{G} sunspot number. In this sense we assert that these simple lagged relationships should be avoided as a valid method to predict the following solar activity amplitude.

1. Introduction

Solar activity prediction has acquired in recent decades a more prominent status within the scientific community, mostly as a consequence of the potential large impact of solar activity in human technological activity (spacecraft, power grids, human activity in space, *etc.*). This interest covers very different temporal scales, ranging from the prediction of short-term space weather events (a few hours or days in advance) to the long-term forecasts on the magnitude of the following solar cycle (*i.e.*, several years in advance). The short-term prediction is usually related to the effects of solar disturbances (*e.g.*, solar flares or large Coronal Mass Ejections, CMEs) that can disturb power grid networks and pose large risks to spacecraft and high radiation hazards to astronauts in orbit on long-term temporal scales. In recent years several solar monitoring satellites (*e.g.*, SOHO and *Coriolis*) have shown their ability to predict the timing for the arrival to earth of such CMEs, enabling humans to protect spacecraft, power grids, and humans in space (Baker, 2000; Wu *et al.*, 2000).

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Long-term predictions of the solar cycle have become relatively popular in the past two decades and are usually based on characteristics of sunspot activities in successive cycles. The sunspot activity that would characterise the recent cycle 23 was the object of intense forecast predictions (summarised in Obridko, Oraevsky, and Allen, 1994) and many of these different prediction methods were later evaluated (Joselyn *et al.*, 1997). Similarly, for the next solar cycle 24, several predictions have been published in recent years using a large variety of methods (*e.g.*, Kane (2007) reviewed a comprehensive list of predictions). When we look at the entire ensemble of predictions, the large spread of values predicted for the cycle 24 peak becomes striking. The majority of these predictions fall in the range 110–130 for the smoothed sunspot number. However, very high values – in the ranges 155–180 and 135–185 – were obtained, respectively, by Dikpati, de Toma, and Gilman (2006) using a particular dynamo model and by Hathaway and Wilson (2006) using a particular geomagnetic precursor technique model. In contrast, very low values were also obtained using prediction methods based on the strength of the polar fields for the current cycle by Schatten (2005) and Svalgaard, Cliver, and Kamide (2005) in the ranges 50–110 and 67–83, respectively. This large spread of results reflects, at least to a major extent, the relative infancy of many of the models employed for this purpose.

A simple approach to predict the next solar cycle amplitude, devised more than half a century ago by Chernosky (1954), established the relationship between the length of the solar cycle (SCL_n) and the amplitude of the next cycle (SCA_{n+1}). This relationship has been reassessed in more detail in recent years by numerous authors such as Hathaway, Wilson, and Reichmann (1994, 2002), Wilson, Hathaway, and Reichmann (1996), Solanki *et al.* (2002), and Hathaway and Wilson (2005). In this sense, the recent work by Kane (2008) should be regarded as the last effort to use this technique to predict the following cycle (24) amplitude.

We should emphasise that SCL time series have been used in a totally different context, namely as a proxy of long-term solar activity level, by using different measurements and definitions owing to its speculative relation with climate (Friis-Christensen and Lassen, 1991; Laut, 2003; Damon and Laut, 2004). Different methods to evaluate SCL provide values lightly different (Mursula and Ulich, 1998; Benestad, 2005; Vaquero, García, and Gallego, 2006). Here, we have used the standard 13-month smoothed monthly R_{G} values and SCL as defined by the time interval between subsequent minima.

The main objective of this paper is to study the evolution of the relationship between SCL_n and SCA_{n+1} by using sunspot numbers during the past 250 years approximately to verify the utility of this relationship for the purpose of forecasting the solar cycle amplitude.

2. Evolution of the Relationship between SCL_n and SCA_{n+1}

Kane (2008) compared the length of a solar cycle against the maximum amplitude for the following cycle, finding an anticorrelation statistically significant ($r = -0.68$), although the inferred standard error of estimate obtained was acknowledged to be relatively large. Removal of cycle pairs 15/16, 19/20, and 20/21 (statistical outliers) yields a regression that is highly statistically significant ($r = -0.85$) and reduces the standard error of estimate by 18%. Based on this approach the SCA for cycle 24 was estimated by Kane (2008) to be about 94 ± 44, using the smoothed values of the International Sunspot Number or R_{Z} (Clette *et al.*, 2007).

During the past decade, some authors have concluded that the reconstruction of solar activity based on counting the number of sunspot groups (R_{G}) made by Hoyt and Schatten (1998a, 1998b) can be more useful with respect to the standard time series of R_{Z}. In

Table 1 Data used in this study including date of minimum and SCL and SCA for solar cycles 1 – 22 calculated with both International and Group Sunspot Numbers.

Solar cycle	International Sunspot Number			Group Sunspot Number		
	Date of minimum	SCL_n	SCA_{n+1}	Date of minimum	SCL_n	SCA_{n+1}
1	February 1755	135	112.6	January 1755	130	71.0
2	May 1766	109	158.7	April 1766	110	78.2
3	June 1775	111	141.2	June 1775	106	90.1
4	September 1784	163	49.2	April 1784	187	51.1
5	April 1798	152	48.7	November 1799	128	31.5
6	December 1810	149	71.6	July 1810	153	64.4
7	May 1823	126	147	April 1823	130	116.8
8	November 1833	121	132.3	February 1834	113	93.2
9	December 1843	143	98.2	July 1843	149	85.8
10	November 1855	135	140.7	December 1855	137	99.9
11	February 1867	139	75.7	May 1867	139	68.2
12	September 1878	137	88.7	December 1878	134	96
13	February 1890	143	63.5	February 1890	143	64.6
14	January 1902	137	105.7	January 1902	138	111.3
15	August 1913	121	78.2	July 1913	121	81.6
16	July 1923	122	120.7	August 1923	121	125.1
17	September 1933	125	151.9	September 1933	126	145.2
18	February 1944	122	202.7	March 1944	121	186.1
19	April 1954	126	110.9	April 1954	122	109.3
20	October 1964	139	164.8	June 1964	141	154.2
21	March 1976	124	157.8	March 1976	120	153.5
22	September 1986	115	121.5	March 1986	n.a.	n.a.

particular R_G has been considered more robust for use in extending the sunspot cycle data further back in time and thereby adding more cycles and improving the statistics (Hathaway, Wilson, and Reichmann, 2002; Usoskin and Kovaltsov, 2004; Vaquero, 2007). Thus, R_G numbers are strongly recommended for analysis of sunspot activity prior to 1880, although the R_Z numbers are regarded to be slightly more useful for characterising the recent levels of solar activity (Hathaway, Wilson, and Reichmann, 2002).

We have calculated all the relevant parameters used by Kane (2008) using the R_G numbers and the standard methodology. The results, listed in Table 1, include the date of minimum and SCL and SCA for solar cycles 1 – 22 calculated with both sunspot numbers. Although the R_G numbers can be used from 1610, we have restricted the use of this index from 1755 (solar cycle number 1) to allow for a more straightforward comparison of our study with Kane (2008), but also to avoid the large gap of sunspot data during the years 1742 – 1745 (Vaquero, Gallego, and Sánchez-Bajo, 2007). In any case, we must acknowledge that there are still some gaps present in the R_G series, especially during the years 1777 – 1794 (Vaquero, 2007). We have used a simple linear interpolation to fill these gaps (monthly scale) because the majority of such gaps have the minimal duration (one month).

We have computed and represented the correlation coefficient between SCL_n and SCA_{n+1} using running windows of 5 and 11 solar cycles, based on both the R_Z (Figure 1) and R_G (Figure 2) series. The cycle number represented in the abscissa of all four panels

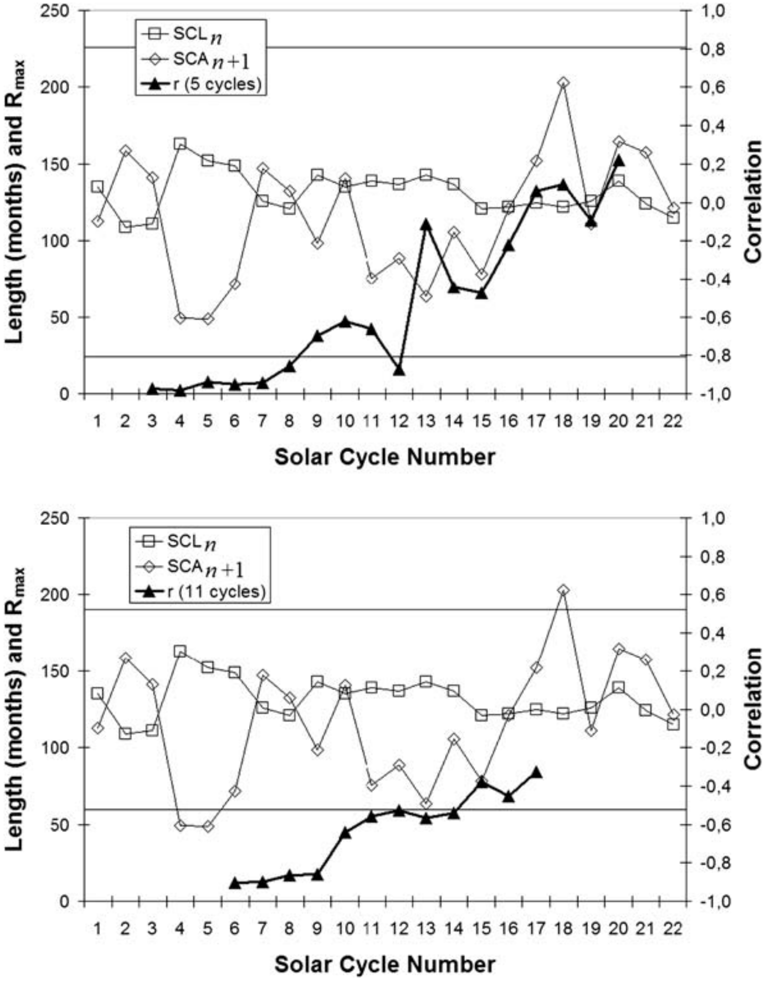


Figure 1 Evolution of the correlation coefficient between SCL$_n$ and SCA$_{n+1}$ for running windows of 5 (upper panel) and 11 (bottom panel) solar cycles.

shown in both figures refers to the mid-point considered for the amplitude cycle. Furthermore, to highlight the periods that are characterised by significant correlation values, the two horizontal lines represent the 10% statistical significance level. This significance threshold level corresponds to $|r| > 0.52$ for the 11-year running window (upper panels in Figures 1 and 2) and increases substantially to $|r| > 0.81$ when only five points are used to compute each correlation value (lower panels in Figures 1 and 2).

Results obtained with R_Z (Figure 1) show a remarkable decline in the strength of the relationship between SCL$_n$ and SCA$_{n+1}$ (*i.e.*, a lack of stationarity in this association, independently of the running-window length applied). The trend and interannual evolution obtained with the five-cycle window is larger than the corresponding evolution obtained with the longer window. In particular the earlier results obtained for windows centred between cycles 3 and 7 achieved extremely high anticorrelation values ($r < -0.9$). These are followed by an almost continuous decline that soon drops below the nonsignificant threshold

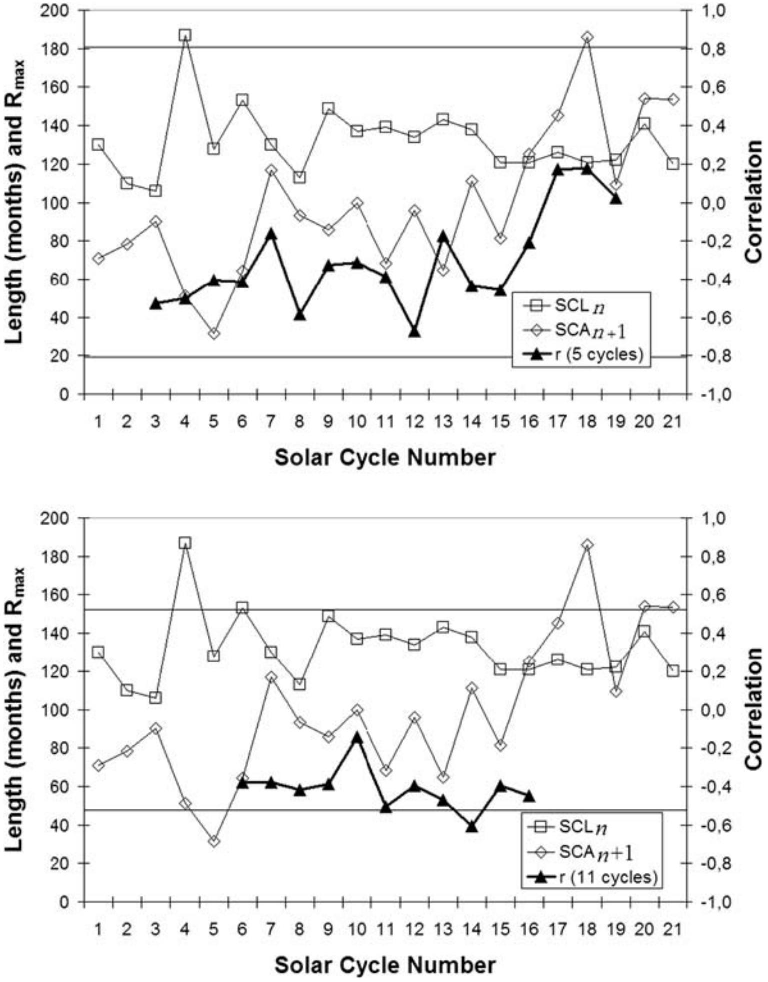
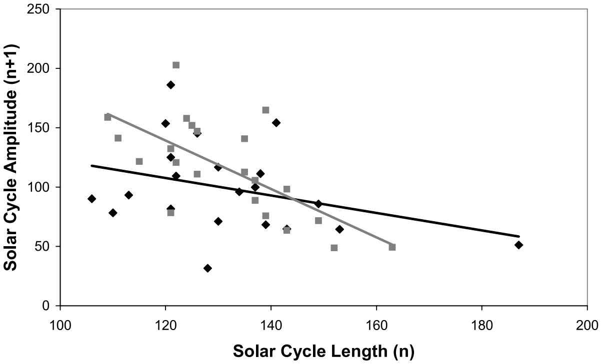


Figure 2 Evolution of the correlation coefficient between SCL_n and SCA_{n+1} for running windows of 5 (upper panel) and 11 (bottom panel) solar cycles using Group Sunspot Number.

for windows centred between cycles 9 and 20, with the exception of one window centred in cycle 12 (Figure 1, upper panel). This temporal evolution is very similar to what was obtained by Solanki *et al.* (2002), although in their case a six-cycle moving window was applied (see their Figure 4). The corresponding results obtained with the 11-cycle running window (Figure 1, lower panel) reveals a larger inertia of the interannual changes. In any case, the temporal evolution corresponds basically to the smoothed version of the five-cycle window analysis, with a continuous decline between high values of anticorrelation obtained for windows centred in cycles 6 – 10 ($r < -0.8$); nevertheless in this case, only the last three windows present nonsignificant values.

A similar analysis applied to the R_{G} time series reveals a different picture, albeit the decreasing trend that also characterises the five-cycle running window (Figure 2). In particular the five-cycle running window presents much lower correlation values at the earlier section and does not reach the significance threshold for any subperiod analysed (Figure 2, upper

Figure 3 Linear relationship between amplitude of solar cycle $n+1$ and length of solar cycle n for R_Z (grey colour) and R_G (black colour) numbers.



panel). However, the analysis with the 11-cycle running window displays a more constant temporal evolution (Figure 2, lower panel), unlike what could be observed with the R_Z series (Figure 1, lower panel). In this case the correlation values observed for the windows centred between cycles 11 and 16 are close or slightly below the 10% significance threshold (window centred in cycle 14).

From a global point of view, we have obtained a correlation coefficient equal to −0.682 for the 22 solar cycles using R_Z (statistically significant at the 10% level) and −0.342 for 21 solar cycles using R_G (though not statistically significant at the 10% level). The corresponding linear regression lines are represented in Figure 3, where one can appreciate the larger dispersion of data points when the R_G series is used. The mathematical expressions for these linear relationships can be written as

$$\mathrm{SCA}_{n+1} = (196.01 \pm 61.70) - (0.74 \pm 0.46)\mathrm{SCL}_n$$

and

$$\mathrm{SCA}_{n+1} = (383.94 \pm 64.69) - (2.04 \pm 0.49)\mathrm{SCL}_n$$

for R_G and R_Z numbers, respectively.

3. Conclusions

Solanki *et al.* (2002) indicated that the solar cycle length record precedes the solar cycle amplitude although the exact relationship might be complex and nonstationary. This result implies that the solar dynamo retains a type of memory from one cycle to the next. Overall our results obtained with R_Z are compatible with the findings attained by Solanki *et al.* (2002). However, we are convinced that the use of the standard sunspot index R_Z prior to 1880 can be misleading for long-term analysis such as the one performed here. In fact, it must be emphasised that the correlation values obtained by using the R_G series are considerably lower than those achieved with the corresponding R_Z index, particularly for the first half of the entire record. In fact, as shown before the results obtained with R_G are relatively poor from the statistical point of view. A similar fact has been detected recently by Dikpati, Gilman, and de Toma (2008) for the well-known "Waldmeier effect" (anticorrelation between the peak in sunspot number of a cycle and the time from minimum to reach that peak). Thus, the correlation between cycle rise time and cycle peak is $r = -0.71$ for R_Z, $r = -0.34$ for R_G, and $r = 10^{-4}$ for sunspot area.

Another interesting possibility that could distort the relation between SCA and the preceding SCL is related to the possible existence of a "lost" solar cycle, a theory proposed by Usoskin, Mursula, and Kovaltsov (2001). According to these authors, solar cycle number 4 (1784 – 1793) could be considered as a normal 10-year-long cycle between 1784 and 1793 followed by a short and weak cycle in 1793 – 1800. This proposal has been further developed in subsequent works (*e.g.*, Usoskin, Mursula, and Kovaltsov, 2002; Krivova, Solanki, and Beer, 2002; Usoskin, Mursula, and Kovaltsov, 2003). We would like to stress that in recent years we have used new archival data to modify slightly the sunspot number during the years 1784 (Vaquero, 2004), 1785 (Vaquero, Trigo, and Gallego, 2005), and 1786 (Vaquero *et al.*, 2007). Additional information on solar activity around the year 1793 could provide useful insight to further clarify this controversial problem.

Therefore, one should be very cautious when using the correlation between SCA and the preceding SCL as the basis to predict the next solar cycle amplitude. For example, Kane (2008) used this approach to estimate a SCA for the current solar cycle 24 of about 94 ± 44. The large range of potential values obtained by Kane (2008) corresponds to the 90% prediction interval using R_Z based on a linear relationship that excludes several points considered as "outliers." However, the author excluded some of the most recent cycles of solar activity, namely cycles 15, 18, and 20.

The results described in the previous section indicate that this relationship is only strongly significant in a statistical sense during the first half of the historical record of R_Z sunspot number. However, it is considerably less significant for the R_G sunspot number, an index that has been shown to be more appropriate for studying the solar activity during historical times (Usoskin and Kovaltsov, 2004; Vaquero, 2007). In this sense we assert that these simple lagged relationships and other similar procedures should not be considered as a valid method to predict the following solar activity amplitude.

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